Module IV

Push Down Automata (PDA)

Context ~~ter~~ free languages are recognised using pushdown automata. It is a finite state machine with the addition of a stack.

Components of PDA

PDA has a read only i/p tape which consists of i/p alphabet, a finite cntrl, a set of final states and an initial state as in the case of FA. In addition to these, it ~~a~~ has a stack called the Pushdown store (PDAS). It is a read write PDS, as we add elements to PDS or remove elements from PDS.

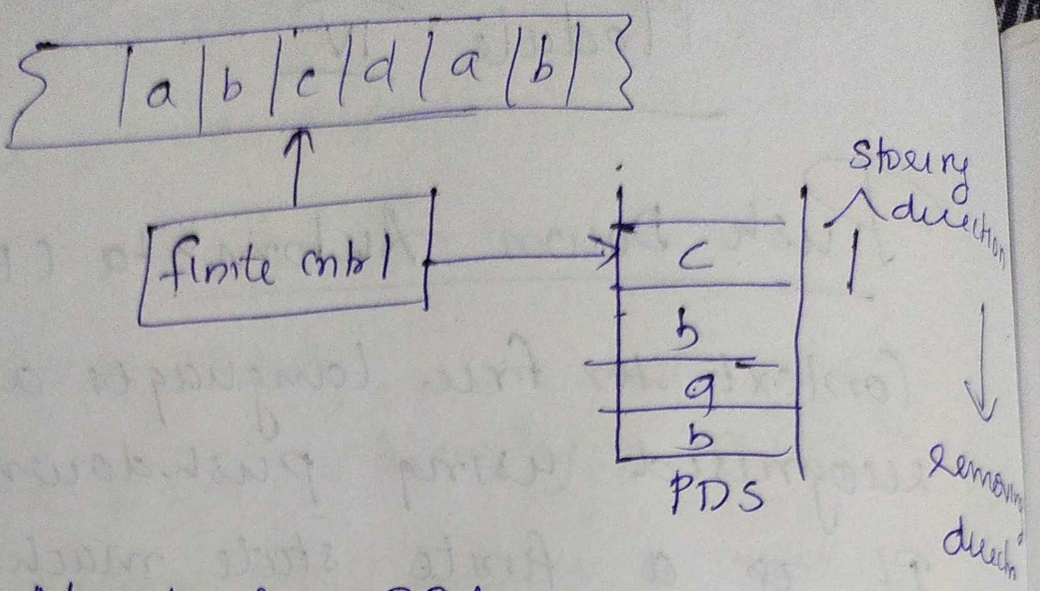


fig: Model of a PDA.

A FA is in some state i.e reading an i/p symbol moves to a new state. The PDA is also in some state i.e on reading an i/p symbol & the top most symbol in PDS, it moves to a new state i.e writes a string of symbols in PDS.

Formal defenition of PDA

(Non-Deterministic PDA (NPDA))

A PDA consists of 7 tuple structure namely $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Q = finite non-empty set of states

Σ = Finite non-empty set of i/p alphabets.

Γ = A finite stack alphabet i.e; set of symbols that are allowed to push onto the stack.

q_0 = Start state

Z_0 = Start symbol on PDS (stack)

F = Set of accepting or final states

δ = A transition in which maps

~~$Q \times (\Sigma \cup \{\epsilon$~~

$Q \times (\Sigma \cup \{\epsilon\}) \times (\Gamma \cup \epsilon) \longrightarrow Q \times \Gamma^*$

eg:- $\delta(q, a, x) = (p, \gamma)$

Here q is a state in Q, a is an i/p symbol in Σ or $a = \epsilon$. X is a stack symbol.

P is a new state, γ is a string of stack symbols that replaces x at the top of stack. If $\gamma = \varepsilon$, then stack is popped; if $\gamma = x$, then the stack is unchanged, and if $\gamma = Yz$, then x is replaced by z and Y is pushed onto stack.

Non-Deterministic PDA (NPDA)

In Non-deterministic PDA, the transition fn. is as follows

$$Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \longrightarrow Q \times \Gamma^*$$

NPDA having some finite no of choices of moves in each situation. The moves will be of 2 types.

1) In the 1st type of move, an i/p symbol is used. Depending on the i/p symbol, the top symbol on the PDS i.e state of finite cntrl, a no of choice are possible.

$$\delta(q, a, z) = \{(P_1, \gamma_1), (P_2, \gamma_2) \ldots (P_m, \gamma_m)\}$$

where q & P_i, $1 \le i \le m$ are states $a \in \Sigma$, $z \in \Gamma$, & γ_i is in Γ^*, $1 \le i \le m$

The above notation indicates that PDA in state q with i/p symbol 'a' and 'z', the top element on the stack, can for any i, enter the state P_i, replace the symbol z by string γ_i & advances the i/p head one symbol.

2) Second type of move (called ϵ-move) is similar to the first, except that the i/p symbol is not used and the i/p ~~hea~~ head ~~as~~ is not advanced after the move. This type of the move allows the PDA to manipulate the stack without reading i/p symbol.

$\delta(q, \varepsilon, z) = (q, \varepsilon)$

+ A PDA is said to be non-deterministic if 1) $\delta(q, a, b)$ may contain multiple elements or
2) if $\delta(q, \varepsilon, b)$ is not empty, then $\delta(q, c, b)$ is not empty for some i/p symbol c.

Deterministic PDA

A PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is said to be deterministic, if it is an automata subject to restrictions that for every $q \in Q$, $a \in \Sigma \cup \{\varepsilon\}$ & $b \in \Gamma^*$

1. $\delta(q, a, b)$ contains atmost one element. This condition requires that for any given i/p symbol & any stack top, almost one move can be made.

2. If $\delta(q, \varepsilon, b)$ is not empty,
then $\delta(q, c, b)$ must be empty
for every $c \in \Sigma$.

~~Sec~~
second condition is that when an
ε-move is possible for some configuration,
no i/p consuming alternative is available.

Instantaneous Description (ID)

We define an ID to formally
describe the configuration of PDA at
any given instant.

Let $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA.
An instantaneous description (ID) is
(q, x, α) where $q \in Q$, $x \in \Sigma^*$ and $\alpha \in \Gamma^*$

$q \rightarrow$ present state

$x \rightarrow$ i/p string to be read

$\alpha \rightarrow$ present stack configuration.

eg:- $(q, a_1 a_2 \ldots a_n, Z_1 Z_2 \ldots Z_m)$ is an ID

here q is the current state.

$a_1a_2 \ldots a_n$ is the i/p string to be processed

$z_1z_2 \ldots z_m$ is the ~~~~ current string of stack symbols with z_1 at the top of stack & z_m at the bottom.

A move from one ID to another ID will be denoted by the symbol $\vdash$ (move relation)

eg:- $(q, a_1a_2 \ldots a_n, z_1z_2 \ldots z_m) \vdash$
$(q', a_2a_3 \ldots a_n, \beta z_2 \ldots z_m)$

$\beta \rightarrow$ new stack symbol $\in \Gamma^*$

~~Desgning Push Down Automata~~

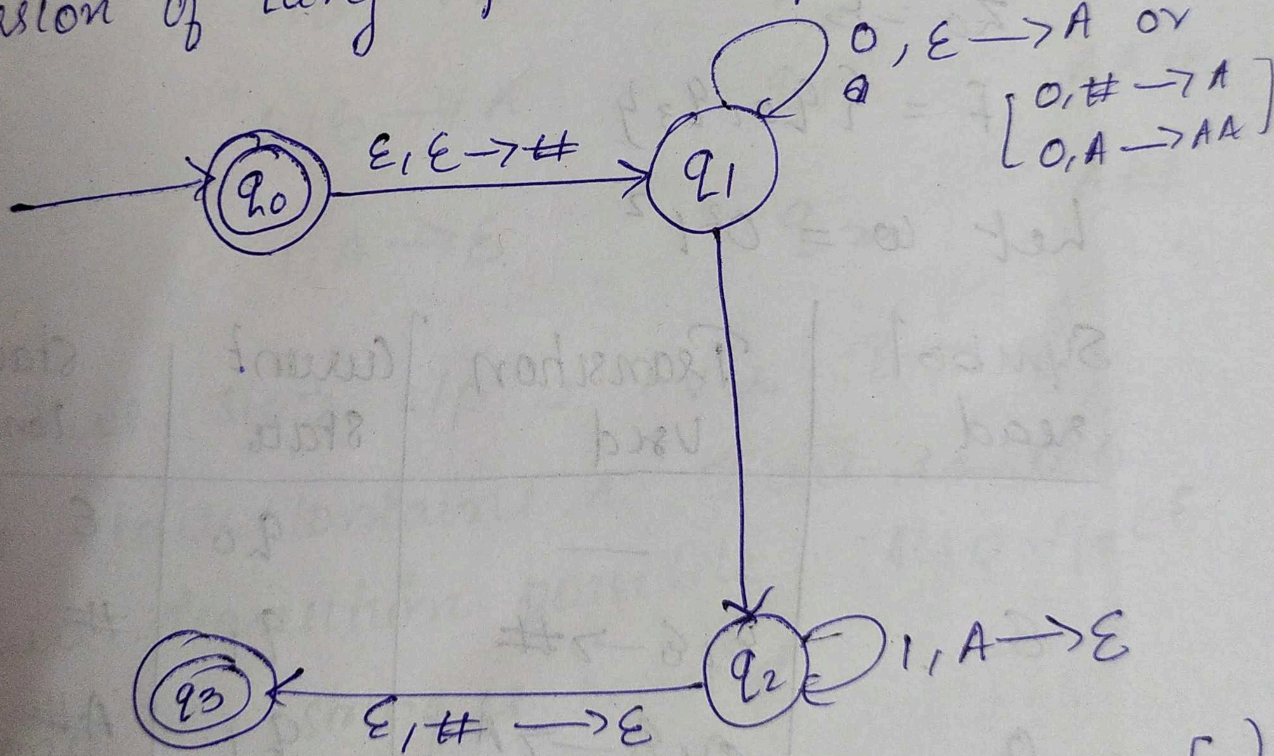
~~1) Design~~

Designing Push down Automata.

1) Design a PDA for $L = \{0^n1^n / n \geq 0\}$

→ The idea is when i/p symbol 0 is read. It will push some symbol say A, onto stack. The moment it sees 1, it moves to some other state & ~~kepp~~ keep popping A with each 1 encountered.

Finally when the i/p is completely read, if the stack becomes empty (i.e., correct i/p is given - stack becomes empty). PDA is the final & correct version of language L accepted.



Thus PDA is $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

here $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{A, B\}$

$\delta(q_0, \varepsilon, \varepsilon) = (q_1, \#)$

$\delta(q_1, 0, \varepsilon) = (q_1, A)$

$\delta(q_1, 0, A) = (q_1, AA)$

$\delta(q_1, 1, A) = (q_2, \varepsilon)$

$\delta(q_2, 1, A) = (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, \#) = (q_3, \varepsilon)$

$q_0 = \{q_0\}$

$z_0 = \varepsilon$

$F = \{q_0, q_3\}$

Let $\omega = 0^2 1^2$

Symbol read	Transition used	Current state	Stack content.
—	—	q_0	ε
ε	$\varepsilon, \varepsilon \rightarrow \#$	q_1	$\#$
0	$0, \varepsilon \rightarrow A$	q_1	$A\#$
0	$0, \varepsilon \rightarrow A$	q_1	$AA\#$
1	$1, A \rightarrow \varepsilon$	q_2	$A\#$
1	$1, A \rightarrow \varepsilon$	q_2	$\#$
ε	$\varepsilon, \# \rightarrow \varepsilon$	q_3	ε

stack is empty & final state q_3 is reached after reading i/p $0^2 1^2$.

Let $w = 0^2 1$

symbol read	Transition Used	current state	stack content
–	–	q_0	ε
ε	$\varepsilon, \varepsilon \to \#$	q_1	$\#$
0	$0, \varepsilon \to A$	q_1	$A\#$
0	$0, \varepsilon \to A$	q_1	$AA\#$
1	$1, A \to \varepsilon$	q_2	$A\#$

A this step i/p is over. Top of the stack contains A. There is no other transition possible. Also $q_2 \notin F$.

Hence the string is rejected.

Equivalence of Acceptance by final state & Empty stack in PDA

There are 2 approaches for accepting i/p.

(1) Acceptance by final state
(2) Acceptance by Empty stack.

These two methods are equivalent i.e if ~~$~~ a language L has a PDA that accepts it by final state then there should be a PDA for L that accepts it by Empty stack.

Acceptance by Final state

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA. Then L(P) the language accepted by ~~Fina~~ P by final state is $L(P) = \{ w \mid (q_0, w, Z_0) \vdash^{*} (q, \varepsilon, \alpha) \}$ for some state $q \in F$ (final state) and any stack symbol α.

Here stack content is not empty but final state is reached.

Acceptance by Empty Stack

Language accepted by PDA is the set of all i/ps for which some sequence of moves causes the PDA to empty ~~its~~ its stack

We define L(M)., the language accepted by empty stack (or null stack) to be,

for a PDA, $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

we define L(M)

$$L(M) = \{ w \mid (q_0, w, Z_0) \vdash^{*} (P, \epsilon, \epsilon) \text{ fo some } P \in Q \}$$

where (q_0, w, Z_0) is the ~~ent~~ initial ID. This PDA empties the PDS after processing all the symbols of w.

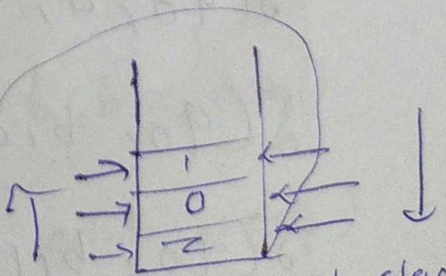
classnotes

Q Design a PDA that accepts the language $L = \{ wcw^R \mid w \in (a+b)^* \}$

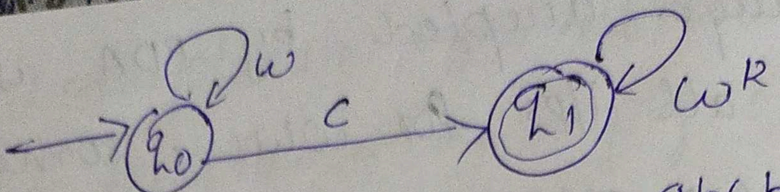
→ $w = ab$

wcw^R = a b c ~~a~~b a ϵ

0 1 nothing pushed to popped



If Z is at stack top, string is accepted



abcba → odd length palindrome

<u>design</u>

$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{0, 1\}$

$q_0 = \{q_0\}$

$z_0 = \{Z\}$

$F = \{q_1\}$

→ when a is read (a is in w) & $q_0 \to q_0$ so 0 pushed

only w... a se 0 pop

only w... only b se 1 pop

Initial symbol can be a or b → $\delta(q_0, a, Z) = (q_0, 0Z)$

→ $\delta(q_0, b, Z) = (q_0, 1Z)$

$\delta(q_0, a, 0) = (q_0, 00)$

$\delta(q_0, a, 1) = (q_0, 01)$

$\delta(q_0, b, 0) = (q_0, 10)$

$\delta(q_0, b, 1) = (q_0, 11)$

~~acceptance by empty stack~~

$\delta(q_0, c, 0) = (q_1, 0)$ — no change in stack top

$\delta(q_0, c, 1) = (q_1, 1)$

only when a ε 0 pop — $\delta(q_1, a, 0) = (q, \varepsilon)$

only when b ε 1 pop — $\delta(q_1, b, 1) = (q, \varepsilon)$

$\delta(q, \varepsilon, z) = (q, \varepsilon)$ — acceptance by empty stack so z should be also poped

$(q_0, abcba, z) \vdash (q_0, bcba, 0z)$

$\vdash (q_0, cba, 10z)$

$\vdash (q_1, ba, \underline{1}0z)$

$\vdash (q_1, a, 0z)$

$\vdash (q_1, \varepsilon, z)$

$\vdash (q_1, \varepsilon, \varepsilon)$

So, i/p string ε stack is empty so abcba is accepted by PDA

$(q_0, abcbb, z) \vdash (q_0, bcbb, 0z)$

$\vdash (q_0, cbb, 10z)$

$\vdash (q_1, bb, 10z)$

$\vdash (q_1, b, 0z)$

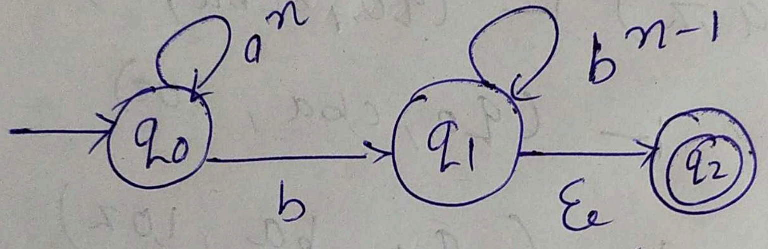
This string is not accepted

23/01/22 classnotes

Q Design a PDA for the language

$L = \{a^n b^n / n \geq 1\}$

→ Transition diagram is :-



Acceptance by F-S

after reaching F-S, string is accepted.

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

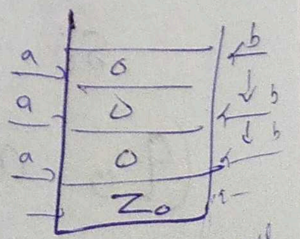
$\Gamma = \{0, z\}$

$q_0 = \{q_0\}$

$Z_0 = \{z\}$

$F = \{q_2\}$

$a^3 b^3$



If ε comes if Z_0 is at stack top, Z_0 can be popped i.e. moving to F.S

$\delta(q_0, a, z) = (q_0, 0z)$

$\delta(q_0, a, 0) = (q_0, 00)$

$\delta(q_0, b, 0) = \cancel{(q_0, 0)}\ (q_1, \varepsilon)$

state change

ε indicates pop

nothing is pushed but popped

$\delta(q_1, b, 0) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z) = (q_2, z)$

no stack top change

aaabbb

$(q_0, aaabbb, z) \vdash (q_0, aabbb, 0z)$

$\vdash (q_0, abbb, 00z)$

$\vdash (q_0, bbb, 000z)$

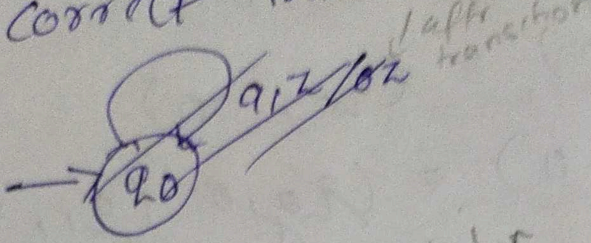
$\vdash (q_1, bb, 00z)$

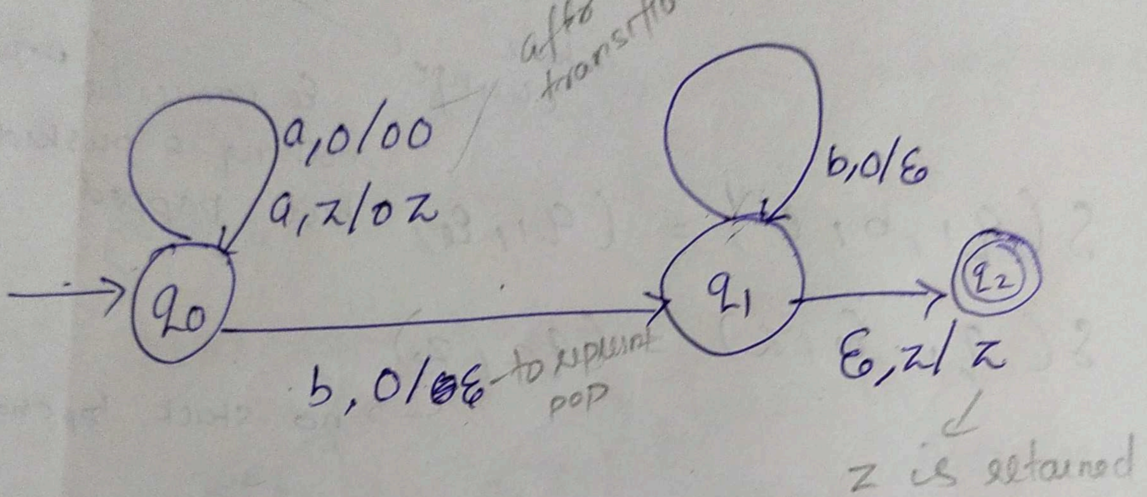
$\vdash (q_1, b, 0z)$

$\vdash (q_1, \varepsilon, z)$

$\vdash (q_2, \varepsilon, z)$

correct transition diagram :-



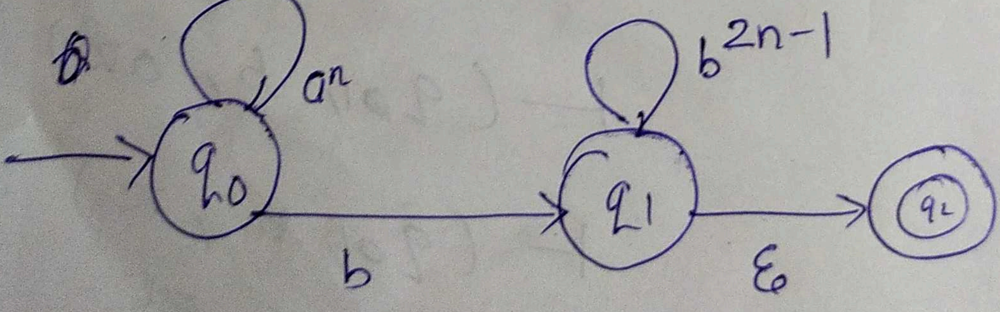


Q Design a PDA for language.

$L = \{ a^n b^{2n} / n > 0 \}$ — acceptance by f.s

→ abb , aabbbb , -

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{0\}$

$q_0 = \{q_0\}$

$z_0 = \{z\}$

$F = \{q_2\}$

$\delta(q_0, a, z) = (q_0, 00z)$

$\delta(q_0, a, 0) = (q_0, 000)$

$\delta(q_0, b, 0) = (q_1, \epsilon)$

$\delta(q_1, b, 0) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z) = (q_2, z)$

abbb

$(q_0, abbb, z) \vdash (q_0, \cancel{a}bb\cancel{b}, 00z)$

$\vdash (q_{\cancel{0}1}, bb\cancel{b}, 0z)$

$\vdash (q_1, b, z)$

$\cancel{\vdash} \; \cancel{(q_1, b)} \; (q_1, b, z)$ is

δ not defined.

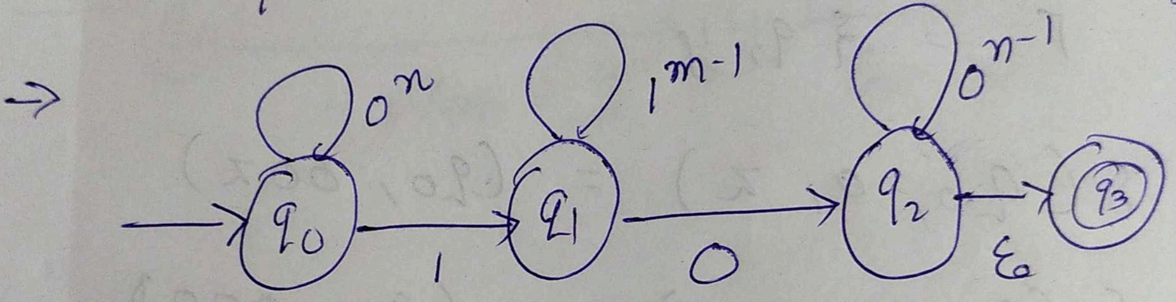
so, abbb $\notin L$.

ie abbb is not accepted by PDA.

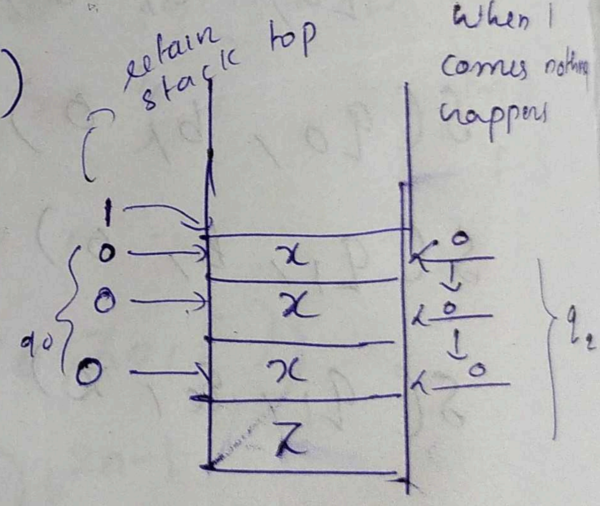
Q $L = \{0^n 1^m 0^n \mid n \geq 1, m \geq 1\}$

Design PDA for L.

Acceptnce by empty stack



$P = (Q, \Sigma, \Gamma, q_0, z_0, F)$



$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$z_0 = \{z\}$

$F = \{q_3\}$

$\delta(q_0, 0, z) = (q_0, xz)$

$\delta(q_0, 0, x) = (q_0, xx)$

$\delta(q_0, 1, x) = (q_1, x)$

$\delta(q_1, 1, x) = (q_1, x)$

$\delta(q_1, 0, x) = (q_2, \varepsilon)$

$\delta(q_2, 0, x) = (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, z) = (q_3, z)$

Consider the string 0011100

$(q_0, 0011100, z) \vdash (q_0, 011100, xz)$

$\vdash (q_0, 11100, xxz)$

$\vdash (q_1, 1100, xxz)$

$\vdash (q_1, 100, xxz)$

$\vdash (q_1, 00, xxz)$

$\vdash (q_2, 0, xz)$

$\vdash (q_2, \varepsilon, z)$

$\vdash (q_2, \wedge, \wedge)$

Here the stack is empty, so the string is accepted by this PDA.

Consider the string, 00110

$(q_0, 00110, z) \vdash (q_0, 0110, xz)$

$\vdash (q_0, 110, xxz)$

$\vdash (q_1, 10, xxz)$

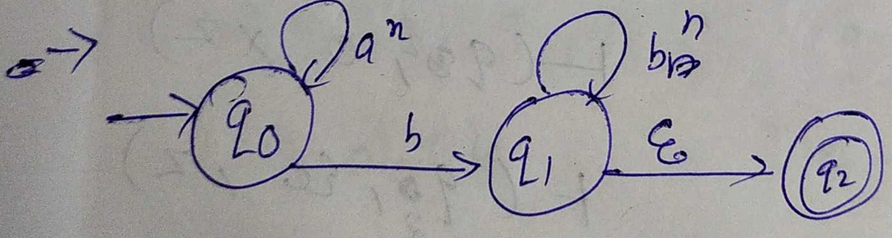
$\vdash (q_1, 0, xxz)$

$\vdash (q_2, \wedge, xz)$

Here $(q_2, \wedge, x) = \phi$ ∴ the string is not accepted.

Q. Consider the PDA for the following language:

$L = \{a^n b^{n+1} \mid n = 1, 2, \dots\}$



Accept by FS

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\Sigma = \{a, b\}$,

$\Gamma = \{0, z\}$

$q_0 = q_0$, $z_0 = z$ $F = \{q_2\}$

$\delta(q_0, a, z) = (q_0, 0z)$

$\delta(q_0, a, 0) = (q_0, 00)$

$\delta(q_0, b, 0) = (q_1, 0)$

$(q_1, b, 0) = (q_1, \epsilon)$

$(q_1, \wedge, z) = (q_2, z)$

Consider the string aabbb

$(q_0, aabbb, z) \vdash (q_0, abbb, 0z)$

$\vdash (q_0, bbb, 00z)$

$\vdash (q_1, bb, 0z)$

$\vdash (q_1, b, z)$

$\vdash (q_1, \epsilon, z)$

$\vdash (q_2, \wedge, z)$

Here q_2 is a final state. So the string aabbb is accepted by this PDA.

Consider the string abbb

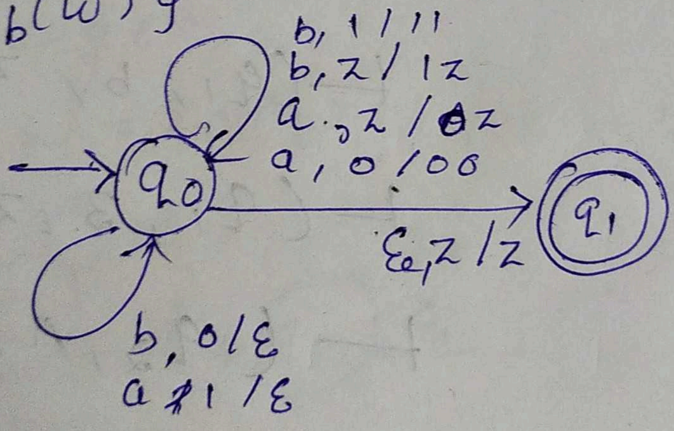
$(q_0, abbb, z) \vdash (q_0, abbb, 0z)$

(q_1, bb, z)

(q_1, b, z)

$\delta(q_1, b, z) = \phi$. so the string abbb is not accepted by the PDA

Q Design a PDA which accepts the language $L = \{ w \in \{a,b\}^* / n_a(w) = n_b(w) \}$



accept by empty stack

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1\}$ $\quad$ $z_0 = z$ $\quad$ $\Gamma = \{a, b, z\}$

$\Sigma = \{a, b\}$ $\quad$ $F = \{q_1\}$

$q_0 = q_0$

$\delta(q_0, a, z) = (q_0, 0z)$

$\delta(q_0, b, z) = (q_0, 1z)$

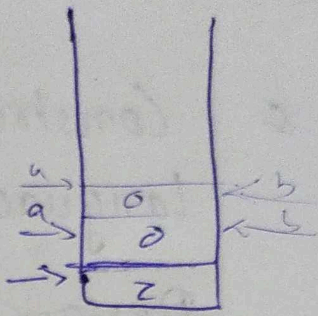
$\delta(q_0, a, 0) = (q_0, 00)$

$\delta(q_0, b, 0) = (q_0, \varepsilon)$

$\delta(q_0, a, 1) = (q_0, \varepsilon)$

$\delta(q_0, b, 1) = (q_0, 11)$

$\delta(q_0, \Lambda, z) = (q_1, z)$



Consider the string baab

$(q_0, baab, z) \vdash (q_0, aab, 1z)$

$\vdash (q_0, ab, z)$

$\vdash (q_0, b, 0z)$

$\vdash (q_1, \Lambda, z)$

Here q_1 is the final state so this string is accepted by the PDA

Consider the string aab

$(q_0, aab, z) \vdash (q_0, ab, 0z)$

$\vdash (q_0, ab, 00)$

$\vdash (q_0, \Lambda, 0z)$

$\delta(q_0, \lambda, 0) = \phi$ so the string is not accepted

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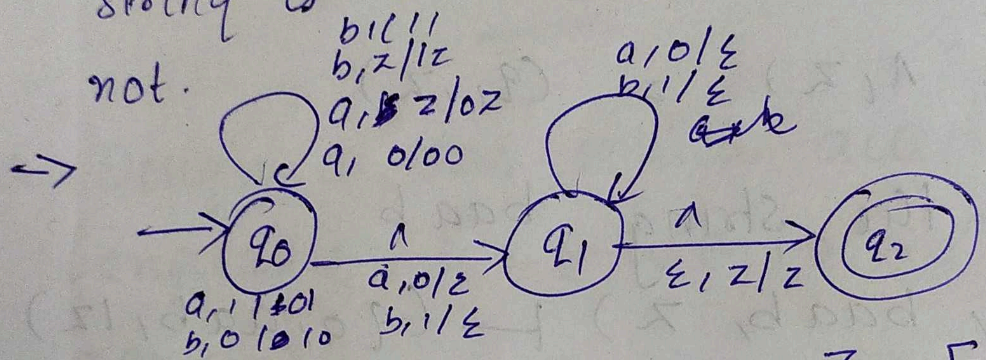
Q. Construct a PDA for accepting the language $L = \{ww^R / w \in \{a,b\}^*\}$. Design an NPDA to CW the given string is even length palindrome or not.



$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\quad Z_0 = Z$

$\Sigma = \{a, b\}$ $\quad q_0 = q_0$

$\Gamma = \{0, 1, Z\}$ $\quad F = \{q_2\}$

$\delta(q_0, a, Z) = (q_0, 0Z)$

$\delta(q_0, a, 0) = (q_0, 00)$

$\delta(q_0, b, Z) = (q_0, 1Z)$

$\delta(q_0, b, 1) = (q_0, 11)$

$\delta(q_0, a, 1) = (q_0, 01)$

$\delta(q_0, b, 0) = (q_0, 10)$

$\delta(q_0, \wedge, 0) = (q_1, 0)$

$\delta(q_0, \wedge, 1) = (q_1, 1)$

$\delta(q_1, a, 0) = (q_1, \wedge)$

$\delta(q_1, b, 1) = (q_1, \wedge)$

$\delta(q_1, \wedge, z) = (q_2, z)$

Consider the string abba

$(q_0, abba, z) \vdash (q_0, bba, 0z)$

$\vdash (q_0, ba, 10z)$

$\vdash (q_1, ba, 10z)$

$\vdash (q_1, a, 0z)$

$\vdash (q_1, \wedge, z)$

$\vdash (q_2, \wedge, z)$

Here q_2 is the final state. So the given string is accepted by this PDA

Consider the string abbb

$(q_0, abbb, z) \vdash (q_0, bbb, 0z)$

$\vdash (q_0, bb, 10z)$

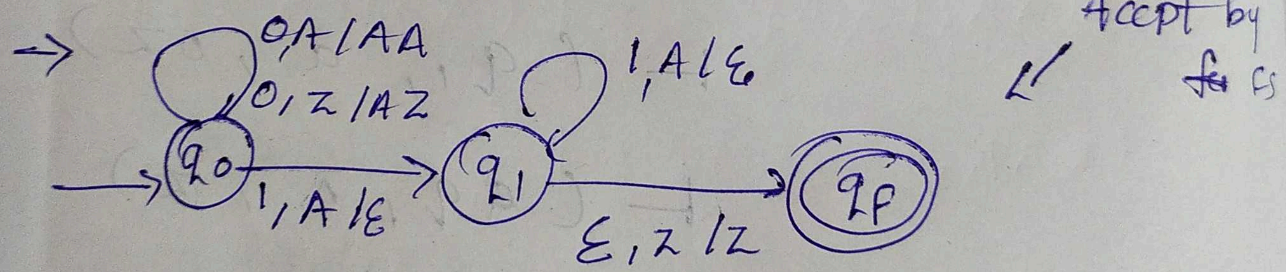
$\vdash (q_1, bb, 10z)$

$\vdash (q_1, b, 0z)$

$\vdash$

$\delta(q_1, b, 0) = \phi$ the string is not accepted.

Q. Design a PDA for $L = \{0^n 1^n / n \geq 0\}$



$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\qquad$ $q_0 = q_0$

$\Sigma = \{0, 1\}$ $\qquad$ $z_0 = Z$

$\Gamma = \{Z, A\}$ $\qquad$ $F = \{q_f\}$

$\delta(q_0, 0, z) = (q_0, \cancel{0}Az)$

$\delta(q_0, 0, A) = (q_0, AA)$

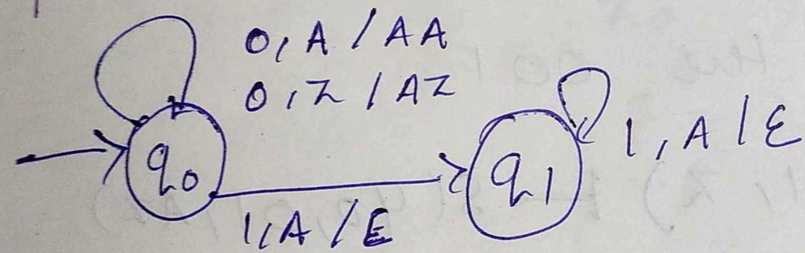
$\delta(q_0, 1, A) = (q_0, \varepsilon)$

~~$\delta(q_0, \varepsilon, z) = ($~~

$\delta(q_1, 1, A) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z) = (q_f, z)$

Acceptance by empty stack ↓



$\delta(q_0, 0, z) = (q_0, Az)$

$\delta(q_0, 0, A) = (q_0, AA)$

$\delta(q_0, 1, A) = (q_1, \varepsilon)$

$\delta(q_1, 1, A) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z) = (q_1, \varepsilon)$

Consider the string 0011

$\delta(q_0, 0011, z) \vdash \delta(q_0, 011, Az)$

$\vdash \delta(q_0, 11, AAz)$

$\vdash \delta(q_1, 1, Az)$

$\vdash \delta(q_1, \varepsilon, z)$

$\vdash \delta(q_f, \varepsilon, z)$

String 0011 is accepted.

Consider the 001

$\delta(q_0, 001, z) \vdash \delta(q_0, 01, Az)$

$\vdash \delta(q_0, 1, AAz)$

$\vdash \delta(q_1, \varepsilon, Az)$

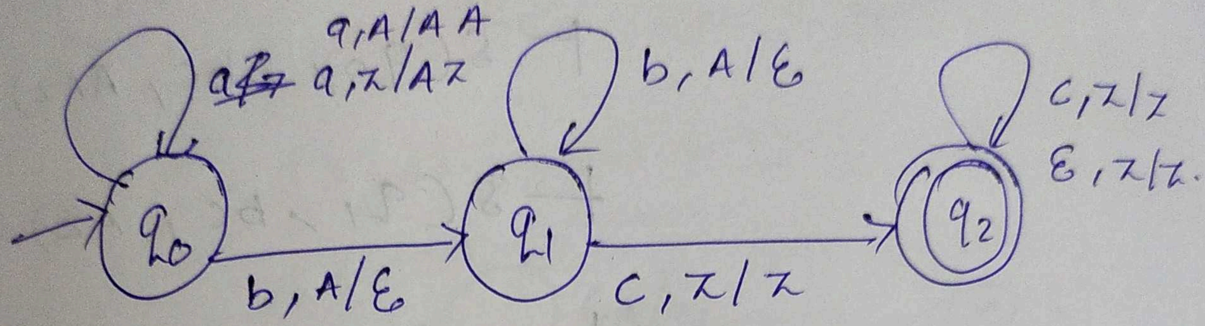
$\vdash$ halt

String not accepted

Q. Design PADA for

By empty Stack

$L = \{a^n b^n c^m \mid n, m \geq 1\}$



$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\quad q_0 = \{q_0\}$

$\Sigma = \{a, b, c\}$ $\quad z_0 = \{z\}$

$F = \{z, A\}$ $\quad F = \{q_2\}$

$\delta(q_0, a, z) = (q_0, AZ)$

$\delta(q_0, a, A) = (q_0, AA)$

$\delta(q_0, b, A) = (q_1, \varepsilon)$

$\delta(q_1, b, A) = (q_1, \varepsilon)$

$\delta(q_1, c, z) = (q_2, Z)$

$\delta(q_2, \varepsilon, z) = (q_2, \varepsilon)$

Consider the string aabbc

$$\delta(q_0, aabbc, z) \vdash \delta(q_0, abbc, Az)$$

$$\vdash \delta(q_0, bbc, AAz)$$

$$\vdash \delta(q_1, bc, Az)$$

$$\vdash \delta(q_1, c, z)$$

$$\vdash \delta(q_2, \varepsilon, z)$$

$$\neq$$

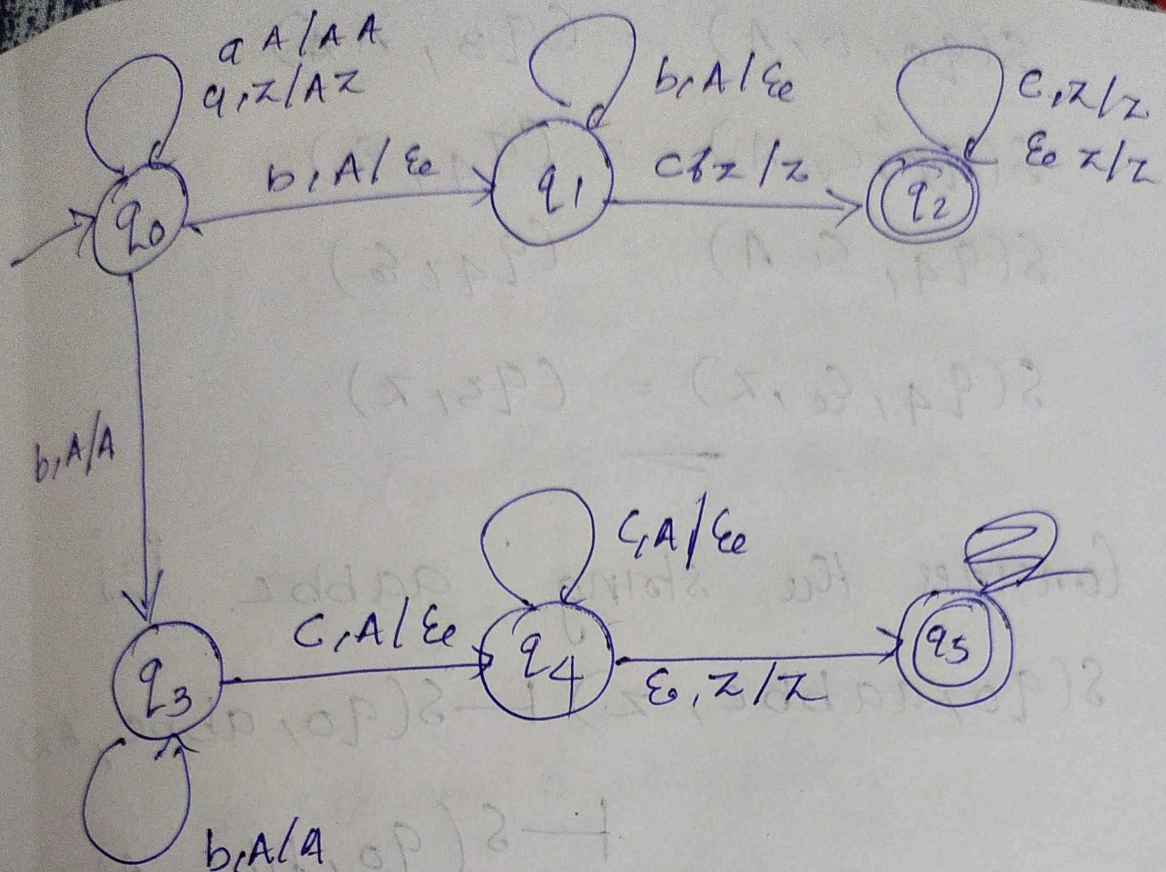
$$\vdash \delta(q_2, \varepsilon, \varepsilon)$$

Here, the string is accepted.

Q. Design a PDA for

$$L = \{ a^i b^j c^k \mid i, j, k > 0 \ \& \ i = j \text{ or } i = k \}$$

→ $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



$Q = \{q_0, q_1, q_2, q_3, q_4, q_5)$

$\Sigma = \{a, b, c)$ $\qquad q_0 = \{q_0\}$

$\qquad z_0 = Z$

$\Gamma = \{A, Z)$ $\qquad F = \{q_2, q_5\}$

$\delta(q_0, a, Z) = (q_0, AZ)$

$\delta(q_0, a, A) = (q_0, AA)$

$\delta(q_0, b, A) = (q_1, \varepsilon)$

$\delta(q_1, b, A) = (q_1, \varepsilon)$

$\delta(q_1, c, Z) = (q_2, Z)$

$\delta(q_2, \varepsilon, Z) = (q_2, \varepsilon)$

$\delta(q_0, b, A) = (q_3, A)$

$\delta(q_3, c, A) = (q_4, \varepsilon)$

$\delta(q_4, c, A) = (q_4, \varepsilon)$

$\delta(q_4, \varepsilon, z) = (q_5, z)$

Consider the string aabbc $i=j$

$\delta(q_0, aabbc, z) \vdash \delta(q_0, abbc, Az)$

$\vdash \delta(q_0, bbc, AAz)$

$\vdash \delta(q_1, bc, Az)$

$\vdash \delta(q_1, c, z)$

$\vdash \delta(q_2, \varepsilon, z)$

$\vdash \delta(q_2, \varepsilon, \varepsilon)$

So, the string is accepted.

Consider the string aabcc $i=j$

$\delta(q_0, \cancel{ab}\, aabcc, z) \vdash \delta(q_0, abcc, Az)$

$\vdash \delta(q_0, bcc, AAz)$

$\vdash \delta(q_3, cc, AAz)$

$\vdash \delta(q_4, c, Az)$

$\vdash \delta(q_4, \epsilon, z)$

$\vdash \delta(q_5, \epsilon, z)$

So the string is accepted.

Push Down Automata & CFG.

For every CFL, there is an NPDA that accepts it & conversely that the language accepted by an NPDA are context free.

Construction of PDA from CFG

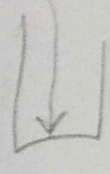
To design a PDA corresponding to a CFG, following are the steps.

Step 1

Let the start symbol of the CFG is S. Then a transition of PDA is

i/p symbol is empty ; initially S is pushed to stack

$$\delta(P, \epsilon, \epsilon) \to (q, S)$$



Step 2

For a prdn of the form $P \to AaB$ a transition of PDA is,

$P \to AaB$

$$\delta(q, \epsilon, P) = (q, AaB)$$

Replace P by AaB i.e RHS

no i/p symbol consider

For prdn of form $P \to a$, a transition of PDA is

$$\delta(q, \epsilon, P) = (q, a)$$

For a prdn of form $P \to \epsilon$, a transition of PDA is

$$\delta(q, \epsilon, P) = (q, \epsilon)$$

step 3

For every terminal symbol a in CFG, a transition of PDA is

$\delta(q, a, a) = (q, \epsilon)$

Thus the PDA is $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where $Q = \{P, q\}$, — only 2 states P & q

Σ = set of terminal symbols in CFG.

Γ = set of terminal & non-terminals in CFG. $q_0 = P$, $F = q$, δ is according to above rules.

Q. Construct a PDA G-equivalent to the following CFG.

$S \rightarrow 0BB$, $B \rightarrow 0S \mid 1S \mid 0$

Test whether 010^4 is in $L(G)$

→ δ is defined as,

Rule1 $\delta(P, \epsilon, \epsilon) = (q, S)$

Rule2 $\delta(q, \epsilon, S) = (q, 0BB)$ ($\because S \rightarrow 0BB$)

$\delta(q, \epsilon, B) = (q, 0S)$

$\delta(q, \epsilon, B) = (q, 1S)$

$\delta(q, \epsilon, B) = (q, 0)$

Rule 3 $\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

String : 010^*

$(p, 010000, \epsilon) \vdash (q, 010000, S)$

$\vdash (q, 010000, 0BB)$

$\vdash (q, 10000, BB)$

$\vdash (q, 10000, 1SB)$

$\vdash (q, 0000, SB)$

$\vdash (q, 0000, 0BBB)$

$\vdash (q, 000, BBB)$

$\vdash (q, 000, 0BB)$

$\vdash (q, 00, BB)$

$\vdash (q, 00, 0B)$

$\vdash (q, 0, B)$

$\vdash (q, 0, 0)$

$\vdash (q, \varepsilon, \varepsilon)$

The string is accepted by PDA

2 Design a PDA for the following CFG $G = (V_N, \Sigma, P, S)$ with $V_N = \{S\}$ $\Sigma = \{(,)\}$ & P is defined as follows

$S \rightarrow SS \mid (S) \mid \varepsilon$

→ $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$P = (\{p, q\}, \{(,)\}, \{(,), S\},$
$\delta, \{p\}, S, \phi)$

δ is defined as follows

$\delta(p, \varepsilon, \varepsilon) = (q, S)$

$\delta(q, \varepsilon, S) = (q, SS)$

$\delta(q, \varepsilon, S) = (q, (S))$

$\delta(q, \varepsilon, S) = (q, \varepsilon)$

$\delta(q, (, () = (q, \varepsilon)$

$\delta(q,),)) = (q, \varepsilon)$

Consider the string $w = ()()$

$(p, ()(), \varepsilon) \vdash (q, ()(), S)$

$\vdash (q, ()(), SS)$

$\vdash (q, ()(), (S)S)$

$\vdash (q,)(), S)S)$

$\vdash (q,)(), ()S)$

$\vdash (q,)(), S)$

$\vdash (q, (), (S))$

$\vdash (q,), S))$

$\vdash (q,),))$

$\vdash (q, \varepsilon, \varepsilon)$

The string is accepted.

3. Design a PDA for the grammar $G = (V_n V_t, P, S)$ where $V_N = \{S\}$ $V_t = \{a, b, c\}$ & production P is defined as $S \rightarrow aSa / bSb / c$.

-> Let PDA be

$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, f)$

$P = (\{p, q\}, \{a, b, c\}, \{S, a, b, c\}, \delta, \{p\}, S, \phi)$

δ is defined as

$\delta(p, \varepsilon, \varepsilon) = (q, S)$

$\delta(q, \varepsilon, S) = (q, aSa)$

$\delta(q, \varepsilon, S) = (q, bSb)$

$\delta(q, \epsilon, S) = (q, c)$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, c, c) = (q, \epsilon)$

Construction of CFG from PDA

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, \phi)$ be a PDA accepting L by empty stack.

Construct $G = (V, T, P, S)$ such that $L(G) = V(M)$

~~Const~~ Here,

$V = \{S\} \cup \{[q, A, p]\}$ where $q, p \in Q$ & $A \in \Gamma$

Rules :-

Rule 1:- For every $q \in Q$, add a prdn

$S \longrightarrow [q_0, z_0, q]$ in prdn set of P of grammar.

start symbol

eg:- If there are 2 states q_0 & q_1,

then $S \longrightarrow [q_0 z_0 q_0] / [q_0 z_0 q_1]$

ie $S \longrightarrow [q_0 z_0 q_0]$

$S \longrightarrow [q_0 z_0 q_1]$

Rule 2

for every $q, r \in Q$, $a \in (\Sigma \cup \varepsilon)$, $x \in \Gamma$.

If $\delta(q, a, x) = (r, \varepsilon)$

Then ~~Eq~~ add a pdn,

$[q x r] \longrightarrow a$

suppose $\delta(q, \varepsilon, x) = (r, \varepsilon)$

Then pdn is :-

$[q x r] \longrightarrow \varepsilon$.

eg:- $\delta(q_0, b, x) = (q_1, \varepsilon)$ &

$\delta(q_1, b, y) = (q_1 \varepsilon)$

Then pdns are:-

$[q_0 x q_1] \rightarrow b$

$[q_1 Y q_1] \rightarrow b$

If $\delta(q_0, \varepsilon, x) = (q_1, \varepsilon)$

then add pdns

$[q_0 x q_1] \rightarrow \varepsilon$

<u>Rule 3</u> : for every $q, r \in Q$,

$a \in \{\Sigma \cup \varepsilon\}$, $x \in \Gamma$ & $x \geq 1$

If $\delta(q, a, x) = (r, x_1 x_2 \ldots x_k)$

where $x_1 x_2 \ldots x_k \in \Gamma$

Then for every choice of $q_1, q_2, \ldots q_k \in Q$, add pdns

$[q\, x\, q_k] \rightarrow a [r\, x_1\, q_1][q_1\, x_2\, q_2][q_2\, x_3\, q_3] \ldots [q_{k-1}\, x_k\, q_k]$

eg: $\delta(q_0, a, x) = (r, AB)$ where $Q = \{q_0, q_1\}$

$[q_0\, x\, q_0] \rightarrow a[r A q_0][q_0 B q_0]$

$[q_0\, x\, q_0] \rightarrow a[r A q_1][q_1 B q_0]$

$[q_0 X q_1] \rightarrow a\,[q A q_0]\,[q_2 B q_1]$

$[q_0 X q_1] \rightarrow a\,[q A q_1]\,[q_1 B q_1]$

Q. find the CFG that generates the same as generated by following NPDA

$M = (\{q_1, q_2\}, \{0,1\}, \{x, z\}, \delta, z_0, q_1, q_2)$

with transitions

$\delta(q_1, 0, z) = (q_1, xz)$

$\delta(q_1, 1, x) = (q_1, xx)$

$\delta(q_1, 0, x) = (q_2, \epsilon)$

→ The corresponding grammar is defined by (V, T, P, S) where

$V = \{S, [q_1 x q_1], [q_1 x q_2], [q_2 x q_1], [q_2 x q_2], [q_1 z q_1], [q_1 z q_2], [q_2 z q_1], [q_2 z q_2]\}$

Productions $S \to [q_0 Z_0 q]$

i) $S \to [q_0, Z q_1]$

$S \to [q_1, Z q_2]$

ii) Pdns corresponding to $\delta(q_1, 0, Z) = (q_1, xZ)$ are written as.

$[q_1 Z q_1] \to 0 [q_1 x q_1] [q_1 Z q_1]$

$[q_1 Z q_1] \to 0 [q_1 x q_2] [q_2 Z q_1]$

$[q_1 Z q_2] \to 0 [q_1 x q_1] [q_1 Z q_2]$

$[q_1 Z q_2] \to 0 [q_1 x q_2] [q_2 Z q_1]$

iii) $\delta(q_1, 1, x) = (q_1 x x)$

$[q_1 x q_1] \to 1 [q_1 x q_1] [q_1 x q_1]$

$[q_1 x q_1] \to 1 [q_1 x q_2] [q_2 x q_1]$

$[q_1 x q_2] \to 1 [q_1 x q_1] [q_1 x q_2]$

$[q_1 x q_2] \to 1 [q_1 x q_2] [q_2 x q_2]$

iv) $\delta(q_1, 0, x) = (q_2, \epsilon)$

$[q_1 x q_2] \longrightarrow 0$

Q Generate CFG for given PDA.
M is defined as $M = (\{q_0, q_1\}, \{0, 1\}, \{x, z\}, \delta, q_0, z, q_1)$
where δ is defined as:

$\delta(q_0, 1, z) = (q_0, xz)$

$\delta(q_0, 1, x) = (q_0, xx)$

$\delta(q_0, 0, x) = (q_0, x)$

$\delta(q_0, \epsilon, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$\delta(q_1, 0, x) = (q_1, xx)$

$\delta(q_1, 0, z) = (q_1, \epsilon)$

→ Variables:-

$V = \{ S, [q_0 x q_0], [q_0 x q_1], [q_1 x q_0], [q_1 x q_1], [q_0 z q_0], [q_0 z q_1], [q_1 z q_0], [q_1 z q_1] \}$

Productions

① $S \longrightarrow [q_0 z q_0]$

$S \longrightarrow [q_0 z q_1]$

② $\delta(q_0, 1, z) = (q_0, xz)$

$[q_0 z q_0] \longrightarrow 1\, [q_0 x q_0][q_0 z q_0]$

$[q_0 z q_0] \longrightarrow 1\, [q_0 x q_1][q_1 z q_0]$

$[q_0 z q_1] \longrightarrow 1\, [q_0 x q_0][q_0 z q_1]$

$[q_0 z q_1] \longrightarrow 1\, [q_0 x q_1][q_1 z q_1]$

3) $\delta(q_0, 1, x) = (q_0, xx)$

$[q_0 x q_0] \longrightarrow 1\, [q_0 x q_0][q_0 x q_0]$

$[q_0 x q_0] \longrightarrow 1\, [q_0 x q_1][q_1 x q_0]$

$[q_0 x q_1] \longrightarrow 1\, [q_0 x q_0][q_0 x q_1]$

$[q_0 x q_1] \longrightarrow 1\, [q_0 x q_1][q_1 x q_1]$

4) $\delta(q_0, 0, x) = (q_0, x)$

$[q_0 x q_0] = 0 [q_0 x q_0]$

$[q_0 x q_1] = 0 [q_0 x q_1]$

5) $\delta(q_0, \epsilon, x) = (q_1, \epsilon)$

$[q_0 x q_1] \longrightarrow \epsilon$

6) $\delta(q_0, \epsilon, x) \longrightarrow (q_1, \epsilon)$

$[q_0 x q_1] \longrightarrow \epsilon$

7) $\delta(q_1, 0, x) = (q_1, xx)$

$[q_1 x q_0] \longrightarrow 0 [q_1 x q_0][q_0 x q_0]$

$[q_1 x q_0] \longrightarrow 0 [q_1 x q_1][q_1 x q_0]$

$[q_1 x q_1] \longrightarrow 0 [q_1 x q_0][q_0 x q_1]$

$[q_1 x q_1] \longrightarrow 0 [q_1 x q_1][q_1 x q_1]$

8) $\delta(q_1, 0, z) = (q_1, \epsilon)$

$[q_1 z q_1] \longrightarrow 0$

Q Convert the following PDA to CFG.

$P(\{q,p\}, \{0,1\}, \{x,z\}, \delta, q, z)$

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$\delta(q,1,z) = (q, xz)$

$\delta(q,1,x) = (q, xx)$

$\delta(q,\varepsilon,x) = (q, \varepsilon)$

$\delta(q,0,x) = (p, x)$

$\delta(p,1,x) = (p, \varepsilon)$

$\delta(p,0,z) = (q, z)$

→ Variables:

~~V = { S, [q x q] [q~~

$V = \{ S, [qxq]\ [qxp]\ [pxq]\ [pxp]$
$[qzq]\ [qzp]\ [pzq]\ [pzp] \}$

1) $S \longrightarrow [qzq]$

$S \longrightarrow [qzp]$

2) $\delta(q, 1, Z) = (q, XZ)$

$[qZq] \longrightarrow 1\,[q\,\overset{X}{\cancel{Z}}\,q]\,[qZq]$

$[qZq] \longrightarrow 1\,[qXp]\,[pZq]$

$[qZp] \longrightarrow 1\,[qXq]\,[qZp]$

$[qZp] \longrightarrow 1\,[qZp]\,[pZp]$

3) $\delta(q, 1, X) = (q, XX)$

$[qXq] \longrightarrow 1\,[qXq]\,[qXq]$

$[qXq] \longrightarrow 1\,[qXp]\,[pXq]$

$[qXp] \longrightarrow 1\,[qXq]\,[qXp]$

$[qXp] \longrightarrow 1\,[qXp]\,[pX\overset{p}{\cancel{q}}]$

4) $\delta(q, \varepsilon, X) = (q, \varepsilon)$

$[qX\overset{q}{\cancel{p}}] \longrightarrow \varepsilon$

5) $\delta(q, 0, X) = (p, X)$

$[qXq] \longrightarrow 0\,[pXq]$

$[qXp] \longrightarrow 0\,[pXp]$

6) $(P, 1, x) = (P, \epsilon)$

$[P \times P] \longrightarrow 1$

7) $\delta(P, 0, z) = (q, z)$

$[P z p] \longrightarrow 0 \, [q \, z \, \overset{p}{\cancel{q}}]$

$[P z q] \longrightarrow 0 \, [q z q]$

02-22